COOPERATION BETWEEN FUZZY AND NONFUZZY TECHNIQUES FOR CYTOLOGICAL IMAGE SEGMENTATION

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Abstract. The present work describes a cooperative approach to segment pictures having unimodal histograms using fuzzy and nonfuzzy entropy. Two procedures are presented allowing the detection of entities in pictures of cells reputed to be difficult to segment.

1 Introduction

The goal of image segmentation is to identify entities in a picture. This task can be achieved by extracting objets from the background. The most commonly used method is the separation of the gray-level histogram into classes using thresholds. The decomposition of each class into connected components separate all entities in the picture. This method is called thresholding.

In the ideal case, thresholds are located in the valleys between two successive peaks of the histogram. In practice, these situations do not often occur because many pictures have either single-peak histograms (see figure 1) or gray-level distributions that are perturbed by noise or texture. For unimodal histograms, a great sensibility in the resulting picture thresholded occurs when the threshold is located close to the mode.

In section 2, some usual techniques for thresholding are presented. Section 3 is dedicated to applications of fuzzy logic in image thresholding. Several distributions of gray-level frequency in the histogram have been proposed in section 4. Our technique does not need any *a priori* knowledge about the picture only the gray-level histogram is required. A parameter to determine the range of uncertainty in the dynamic of gray levels is necessary. A cooperation between fuzzy and non fuzzy techniques is proposed in this section in order to segment pictures of cells. Some methods are implemented and used to threshold several pictures. The results and the robustness of cooperation between fuzzy and nonfuzzy techniques are shown in section 5. Conclusions are given in section 6.

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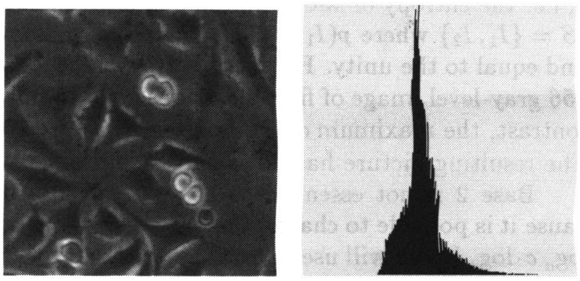


(a) Murine fibroblasts (L929, ATCC, CCL1)

(b) histogramme

Figure 1: (a) An image of fibroblasts observed in phase contrast and (b) its histogram.

2 Classical techniques

Shannon, in 1948, developed the equation measuring the quantity of information (entropy) associated with a finite set $S = \{s_1, s_2, \ldots, s_k\}$ of statistically independent events having probability p_i:

$$H(p_1, p_2, \ldots, p_k) = -\sum_{i=1}^{k} p_i \log_2 p_i \quad (1)$$

where $\sum_{i=1}^{k} p_i = 1$.

Let us consider the frequency histogram of a picture I composed of M pixels coded on N gray levels $(0, 1, ..., N-1)$. Assuming that all gray levels are statistically independent, the entropy of the histogram is represented by (1), where $k = N$, the number of pixels with gray level i is f_i and $p_i = f_i/M$ is the occurrence probability of level i.

2.1 *A posteriori* entropy of the histogram

After separating I into two classes I_1 and I_2 (object(O) and background(B)) by a threshold τ, the entropy of the new image, called *a posteriori* entropy, is:

$$PE(\tau) = -p_{I_1} \log_2 p_{I_1} - p_{I_2} \log_2 p_{I_2} \quad (2)$$

where $p_{I1} = \sum_{i=0}^{\tau} f_i/M$ and $p_{I_2} = \sum_{i=\tau+1}^{N-1} f_i/M$.

PE is a measure of the *a posteriori* quantity of information associated with black and white pixels after the thresholding step.

The maximum of PE is given by the threshold τ which separates I into an equal number of white and black pixels. In this case, $PE(\tau) = -2 \cdot 0.5 \log_2 0.5 = 1$, i.e. the entropy of two equiprobably elements of S ($S = \{I_1, I_2\}$ where $p(I_1) = p(I_2) = 0.5$) is maximal and equal to the unity. For picture 1(a), a 256×256, 256 gray-level image of fibroblasts observed in phase contrast, the maximum of PE is given by $\tau_{PE} = 108$. The resulting picture has not visual significance.

Base 2 is not essential to compute results because it is possible to change the bases using $\log_a b = \log_a c \cdot \log_c b$. We will use natural logarithms without considering the constant $\log_2 e$.

2.2 Adapted entropy of the histogram

Kapur et al [6] proposed to use the normalized gray-level histogram as a gray-level probability distribution. Their method works as follows:

$$D_O \;:\; \frac{p_0}{P_\tau}, \frac{p_1}{P_\tau}, \ldots, \frac{p_\tau}{P_\tau}$$

$$D_B \;:\; \frac{p_{\tau+1}}{1-P_\tau}, \frac{p_{\tau+2}}{1-P_\tau}, \ldots, \frac{p_{N-1}}{1-P_\tau}$$

where $P_\tau = \sum_{i=0}^{\tau} p_i$. The goal is to obtain the maximum information between the object and background distributions in the picture by

$$\max_\tau \left(-\sum_{i=0}^{\tau} \frac{p_i}{P_\tau} \ln \frac{p_i}{P_\tau} - \sum_{i=\tau+1}^{N-1} \frac{p_i}{1-P_\tau} \ln \frac{p_i}{1-P_\tau} \right) \quad (3)$$

For picture 1(a) the threshold provided by adapted entropy technique is $\tau_{AE} = 159$ and the resulting picture is shown in figure 2(a). We can note that light cells and some refringerancy rings in the periphery of cells are detected.

2.3 Other methods

Pun [11] proposed another technique to select a threshold based on the asymmetry of the histogram, using an anisotropy coefficient. Kapur et al [6] gave a similar anisotropic heuristic. Also cross entropy [8] can be used for thresholding. Kittlet and Illingworth [7] assume that the populations are distributed normally with distinct means and standard deviations in order to minimize the average pixels classification error rate. Fisher [5] proposed a multi-thresholding technique which minimizes the sum of gray-level variation in classes.

Evidently these methods do not cover all existing techniques. In order to get results more adapted to pictures having unimodal histogram, fuzzy techniques will be presented and proposed in the next sections.

3 Fuzzy sets and their application to image thresholding

The basic notion of fuzzy sets comes from the idea of partial belonging of one element to several classes at the same time. For more details see [13].

Definition 1 *Let X be the universal set. A fuzzy set A in X is defined by a membership function μ_A which associates for each x in X its grade of possessing the property A in [0,1]:*

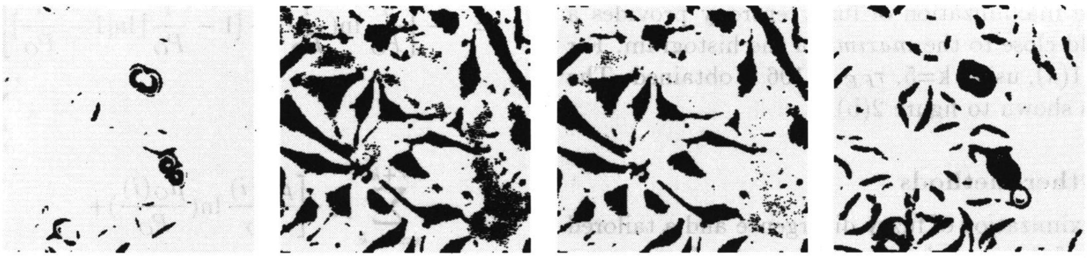
$$\mu_A : X \to [0, 1]. \quad (4)$$

A is a crisp set when $\mu_A = 0$ or 1 for all $x \in X$.

3.1 Interval of ambiguity

At the moment of separating gray levels of the histogram into two classes by a threshold τ, it is possible to consider that some pixels belong to the two classes, mainly for pixels having gray level close to τ.

Fuzzy logic enables to consider the incertainty of membership of gray levels in the interval $[\tau-k, \tau+k]$. In this range, each point belongs fuzzily



(a) (b) (c) (d)

Figure 2: Some results for picture 1(a) using different techniques: (a) adapted entropy: $\tau_{AE} = 159$, (b) fuzzy entropy: $\tau_{FE} = 106$ and new fuzzy entropy: (c) $\tau'_{NFE} = 103$ and (d) $\tau''_{NFE} = 127$.

to two classes, gray levels outside this range are not ambiguous.

Two new problems raise up:

- How to choose the interval $[\tau - k, \tau + k]$?
- In this range how to define the membership of each element to the two classes ?

In order to choose the value of k to achieve a valley in the histogram, Murthy and Pal [9] suggest that the window length $2k$ should be less than the distance between two successive peaks. In order to achieve this result, the hypothesis of convexity of the histogram between the peaks and the existance of a local minimum are assumed. But when the histogram is unimodal, this rule cannot be used.

The second problem can be easily avoided by using the S-function of Zadeh (figure 3), defined by

$$S(x) = \begin{cases} 0 & if \quad x \le a \\ 2[\frac{x-a}{c-a}]^2 & if \quad a \le x \le b \\ 1 - 2[\frac{x-c}{c-a}]^2 & if \quad b \le x \le c \\ 1 & if \quad x \ge c \end{cases}$$

Supposing $a = \tau - k$, $b = \tau$ and $c = \tau + k$, we can use S-function to determine membership function μ_B and $1 - S$-function to determine membership function μ_O. These functions have two essencial particularities: they are symmetrical by reference to τ (crossing over point) and they have not most of their variation which is concentrated in a small interval and a few variation outside (see [9]).

3.2 Fuzzy entropy

One approach that incorporates ambiguous information to define gray-level thresholds in the image histogram is to transform the usual entropy into fuzzy

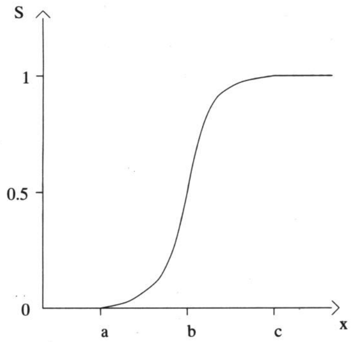


Figure 3: S-function of Zadeh.

logic. De Luca and Termini [4] have shown that incertainty of fuzzy nature in a set $X = \{0, 1, ..., M\}$ can be measured by:

$$FE(X) = \sum_{i=0}^{M} S(x_i) =$$
$$= -\sum_{i=0}^{M} [x_i \ln x_i + (1 - x_i) \ln(1 - x_i)] \quad (5)$$

where S is the Shannon function.

In [10] it is proposed to use equation (5) to measure fuzziness associated with a thresholded image. Then, for each threshold τ there is a range $[\tau - k, \tau + k]$ where the membership of each element to two classes (background and object) belongs to (0,1). As $\ln \mu_O(i) = 0 \;\; \forall i \in [0, \tau - k - 1]$ and $\mu_O(i) = 0 \;\; \forall i \in [\tau + k + 1, N - 1]$, equation (5) becomes:

$$FE(X \backslash \tau, k) = -\sum_{i=\tau-k}^{\tau+k} p_i \cdot \{\mu_O(i) \ln \mu_O(i) + [1 - \mu_O(i)] \ln[1 - \mu_O(i)]\} \quad (6)$$

The maximization of fuzzy entropy provides a threshold close to the *maxima* of the histogram. For picture 1(a), using k=5, $\tau_{FE} = 106$ is obtained. The result is shown to figure 2(b).

3.3 Other methods

The maximization of fuzzy divergence and a tailored version of the probability measure of a fuzzy event can be applied to image thresholding in order to obtain the most ambiguous gray level value in the histogram [1]. Fuzzy c-means (FCM) is an algorithme of fuzzy classification [2, 3]. It can be used to describe a fuzzy classification of gray levels into classes [12].

Evidently these methods do not cover all existing techniques.

4 New fuzzy approaches and cooperation with nonfuzzy techniques

The idea is to use fuzzy entropy but considering the whole dynamic, even outside the interval of uncertainty. Two new methods are presented below. The strength of these methods is their power to provide alternative thresholds.

4.1 Adapted fuzzy entropy

We propose to use membership distributions, where each membership is normalized in its class (before or after τ), defined as follows:

$$D_O \ : \ \frac{\mu_O(0)}{P_O}, \frac{\mu_O(1)}{P_O}, \ldots, \frac{\mu_O(N-1)}{P_O}$$
$$D_B \ : \ \frac{\mu_B(0)}{P_B}, \frac{\mu_B(1)}{P_B}, \ldots, \frac{\mu_B(N-1)}{P_B}$$

where $P_O = \sum_{i=0}^{\tau+k} \mu_O(i)$ and $P_B = \sum_{i=\tau-k}^{N-1} \mu_B(i)$ have the same function of P_τ in section 2.2.

In this case, equation (6) becomes

$$AFE(X\backslash\tau,k) = AFE(O\backslash\tau,k) + AFE(B\backslash\tau,k) =$$
$$= -\sum_{i=0}^{\tau+k} p_i \cdot \left[\frac{\mu_O(i)}{P_O}\ln(\frac{\mu_O(i)}{P_O}) + \right.$$
$$\left. +[1-\frac{\mu_O(i)}{P_O}]\ln[1-\frac{\mu_O(i)}{P_O}]\right]$$
$$-\sum_{i=\tau-k}^{N-1} p_i \cdot \left[\frac{\mu_B(i)}{P_B}\ln(\frac{\mu_B(i)}{P_B}) + \right.$$
$$\left. +[1-\frac{\mu_B(i)}{P_B}]\ln[1-\frac{\mu_B(i)}{P_B}]\right] =$$
$$= -\left[\frac{1}{P_O}\ln(\frac{1}{P_O}) + [1-\frac{1}{P_O}]\ln[1-\frac{1}{P_O}]\right] \cdot$$
$$\cdot \sum_{i=0}^{\tau-k-1} p_i$$
$$-\sum_{i=\tau-k}^{\tau+k} p_i \cdot \left[\frac{\mu_O(i)}{P_O}\ln(\frac{\mu_O(i)}{P_O}) + \right.$$
$$\left. +[1-\frac{\mu_O(i)}{P_O}]\ln[1-\frac{\mu_O(i)}{P_O}]\right]$$
$$-\sum_{i=\tau-k}^{\tau+k} p_i \cdot \left[\frac{\mu_B(i)}{P_B}\ln(\frac{\mu_B(i)}{P_B}) + \right.$$
$$\left. +[1-\frac{\mu_B(i)}{P_B}]\ln[1-\frac{\mu_B(i)}{P_B}]\right]$$
$$-\left[\frac{1}{P_B}\ln(\frac{1}{P_B}) + [1-\frac{1}{P_B}]\ln[1-\frac{1}{P_B}]\right] \cdot$$
$$\cdot \sum_{i=\tau-k}^{N-1} p_i$$

The key point here is when gray levels membership is normalized in each class, elements outside interval of ambiguity are considered. Then, sometimes a more performing thresholding can be detected. Normally, the threshold is found close to the mode.

Adapted fuzzy entropy technique provides $\tau_{AFE} = 108$ for picture 1(a). We can note that there were some changes in comparison with the result given by fuzzy entropy technique but the result does not provide a good semantic interpretation of picture 1(a).

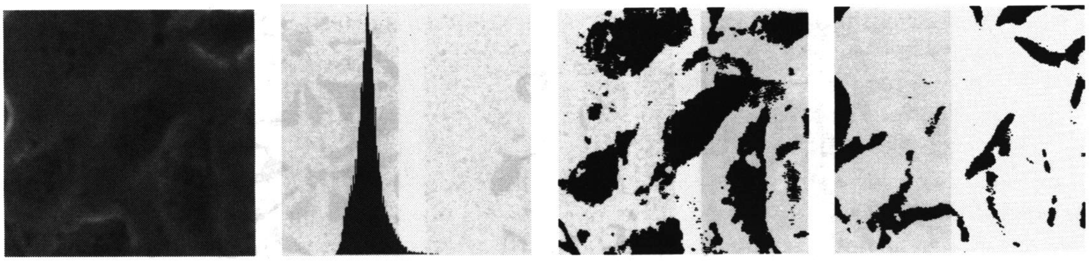
4.2 New fuzzy entropy

Now, let us consider two distributions as follows:

$$D_O \ : \ \frac{\mu_O(0)p_0}{P_O}, \frac{\mu_O(1)p_1}{P_O}, \ldots, \frac{\mu_O(N-1)p_{N-1}}{P_O}$$
$$D_B \ : \ \frac{\mu_B(0)p_0}{P_B}, \frac{\mu_B(1)p_1}{P_B}, \ldots, \frac{\mu_B(N-1)p_{N-1}}{P_B}$$

where $P_O = \sum_{i=0}^{\tau+k} \mu_O(i)p_i$ and $P_B = \sum_{i=\tau-k}^{N-1} \mu_B(i)p_i$.

In this case, the distributions take into account frequency and membership of each gray level associated directly with means of a multiplication and normalized in each class. This association makes possible to compensate low frequencies of gray-levels with strongly belonging to classes and *vice versa*. The measure of information associated with the separa-



(a) (b) (c) (d)

Figure 4: (a) An image of fibroblasts observed in phase contrast, (b) its histogram and some results given by new fuzzy entropy technique: (c) $\tau'_{NFE} = 86$ and (d) $\tau''_{NFE} = 103$.

tion by τ is:

$$NFE(X\backslash\tau,k) = NFE(O\backslash\tau,k) + NFE(B\backslash\tau,k) =$$

$$= -\sum_{i=0}^{\tau-k-1}\left[\frac{p_i}{P_O}\ln\frac{p_i}{P_O} + (1-\frac{p_i}{P_O})\ln(1-\frac{p_i}{P_O})\right]$$

$$-\sum_{\tau-k}^{\tau+k}\left[\frac{\mu_O(i)p_i}{P_O}\ln\frac{\mu_O(i)p_i}{P_O} + \right.$$

$$\left. +[1-\frac{\mu_O(i)p_i}{P_O}]\ln[1-\frac{\mu_O(i)p_i}{P_O}]\right]$$

$$-\sum_{\tau-k}^{\tau+k}\left[\frac{\mu_B(i)p_i}{P_B}\ln\frac{\mu_B(i)p_i}{P_B} + \right.$$

$$\left. +[1-\frac{\mu_B(i)p_i}{P_B}]\ln[1-\frac{\mu_B(i)p_i}{P_B}]\right]$$

$$-\sum_{i=\tau+k+1}^{N-1}\left[\frac{p_i}{P_B}\ln\frac{p_i}{P_B} + (1-\frac{p_i}{P_B})\ln(1-\frac{p_i}{P_B})\right]$$

For picture 1(a), using k=5 and dynamic [64,159], function NFE has two *maxima*: $\tau'_{NFE} = 103$ as shown in figure 2(c) and $\tau''_{NFE} = 127$ which extracts strongly refringence ring in the perifery of cells making possible to detect cells undergoing mitosis and cells which will enter into mitosis (see figure 2(d)). We can note that dark cells are detected.

The key point here is the association of each gray-level with its membership to classes.

4.3 Cooperation between fuzzy and nonfuzzy techniques

When using nonfuzzy techniques we can note that it was not possible to localize a class of dark cells in picture 1(a). But the threshold given by new fuzzy entropy technique, proposed in section 4.2, made it possible to extract dark cells in picture 1(a) after the class of light cells had been eliminated by adapted entropy thresholding. The idea here is to use fuzzy and nonfuzzy entropy in order to obtain classes composing pictures with unimodal histogram, specifically to separate dark cells, background and light cells correctly in pictures of fibroblasts observed in phase contrast. Section 5 presents more results obtained for other pictures.

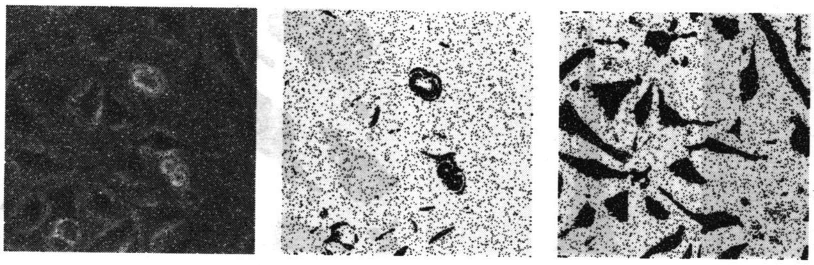
picture	4(a)	5(a)
dynamic	(53,181)	(0,255)
AE	127	153
AC	86	105
ME	128	180
dynamic	(53,127)	(0,153)
FD	min, max	min, max
FE	89	105
AFE	88	102
NFE	86, 103	72

Table 1: Thresholds given by fuzzy and nonfuzzy techniques

5 Implementation, results and comparations

We have implemented some fuzzy and nonfuzzy techniques presented in sections 2 (adapted entropy-AE, anisotropy coefficient-AC and minimum error-ME), 3 (fuzzy divergence-FD and fuzzy entropy-FE) and 4 (adapted fuzzy entropy-AFE and new fuzzy entropy-NFE) using them to threshold other pictures.

Figures 4(a)-(b) show 256×256, 256 gray-level image of fibroblasts observed in phase contrast and its histogram. In order to verify the robustness of the cooperation proposed in section 4.3, picture 1(a) was



(a) (b) (c)

Figure 5: (a) Noisy picture and some results: (b) adapted entropy technique: $\tau_{AE} = 153$ and (c) adapted fuzzy entropy technique: $\tau_{AFE} = 102$.

noised by spot noise with uniform distribution. The histogram is quite similar. Figure 5(a) presents the noisy picture. The last two images have the particularity that their gray-level histograms are unimodal.

Now, we will discuss the results (presented in table 1) obtained from these pictures by the algorithms introduced previously.

For picture 4(a), $\tau_{AE} = 127$ and $\tau_{ME} = 128$ extract slight refringence ring at the perifery of cells caused by their shape. Using the cooperation proposed in section 4.3, i.e. working with [53,127] after [128,181] had been eliminated, $\tau'_{NFE} = 86$ is obtained (this result is given also by the asymmetry coefficient technique) allowing to detect dark cells. Figure 4(c) shows picture 4(a) thresholded by $\tau'_{NFE} = 86$. The new fuzzy entropy technique provides $\tau_{AFE} = 88$. This is not a good result but this threshold is close to $\tau'_{NFE} = 86$; the reason is that close to the mode thresholds can cause many changes in the resulting picture. $\tau''_{NFE} = 103$ extracts strongly refringence ring at the perifery of cells (see figure 4(d)).

Seeing figure 5(b) we notice that $\tau_{AE} = 153$ can eliminate light cells of picture 5(a). Using the cooperation proposed in section 4.3, $\tau_{AFE} = 102$ is the threshold chosen in $[0, 153]$, which permits to extract dark cells (see figure 5(c)).

We note that fuzzy divergence technique is not efficient for unimodal histograms. This measure gives thresholds close to local *maxima* of the histogram, without relation with a semantic interpretation of the picture. Fuzzy entropy gives threshold close to the mode of the histogram due to the great importance given to gray-level probabilities in equation (6). Table 1 shows that, for unimodal histograms, fuzzy divergence technique does not give good results. The size used for ambiguity interval was 10 (k=5). For large values of k local caracteristics loose importance and for little values of k local information is too much reinforced.

6 Conclusion

Fuzzy and nonfuzzy methods for selection of thresholds of a picture have been presented. Several distributions of gray-level frequency (probabilistic and nonprobabilistic) in the histogram enable a cooperation between results given by fuzzy and nonfuzzy entropy in order to obtain suitable thresholds in pictures having unimodal histograms.

We conclude that it is interesting to evaluate results obtained close to the mode in order to choose more adapted thresholds. The alternative results given by the two new techniques proposed permit the efficient extraction of entities in images with unimodal histograms.

The cooperation proposed is robust to spot noise.

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